$$\oint F \cdot dr = \iint \text{curl } F \cdot ds$$

$$\oint F \cdot dr = 0 = \iint \text{curl } F \cdot ds = 0$$

$\text{curl } F = 0$

$\nabla \times F = 0.$

$F = -\nabla V$

Mechanics of a System of Particles

1. External & internal forces.

force on i^{th} Particle is given by

$$F_i = F_i^e + \sum_{j=1}^{N} F_{ij}$$

where F_i^e is external force acting on i^{th} particle due to sources outside the system. F_{ij} is the internal force on the i^{th} particle due to j^{th} particle and the total internal force due to all other particles ($j = 1$ to N) on the i^{th} particle.

According to newton's second law

$$F_i = \dot{P}_i = m_i \frac{dv_i}{dt} = m_i \frac{d^2 r_i}{dt^2}$$

for whole system / for all particles.

$$\frac{d^2}{dt^2} \sum_i m_i r_i = \sum_i F_i^e + \sum_i \sum_{\substack{j \\ i \neq j}} F_{ij}$$

Total external force (F_e)

by Newtons third law

$$F_{ij} + F_{ji} = 0.$$

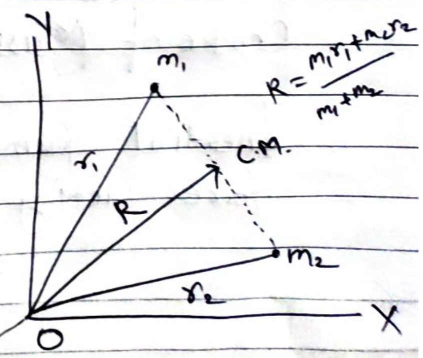
$$F_e = \frac{d^2}{dt^2} \sum_i m_i r_i$$

Centre of Mass of System.

R is position of centre of mass.

$$R = \frac{\sum_i m_i r_i}{\sum_i m_i} = \frac{\sum_i m_i r_i}{M \text{ (total mass)}}$$

$$F_e^e = M \cdot \frac{d^2 R}{dt^2} = M a_{cm}$$



Conservation of Linear Momentum.

differentiation $R = $ —"—

$$M\frac{d\bar{R}}{dt} = m_1\frac{d\bar{r}_1}{dt} + m_2\frac{d\bar{r}_2}{dt} + \cdots m_N\frac{d\bar{r}_N}{dt}$$

$$MV = m_1 v_1 + m_2 V_2 \cdots m_N V_N$$

$$= \sum_{i=1}^{N} m_i v_i$$

$MV = P$ (total linear momentum)

$$\frac{dP}{dt} = \frac{d}{dt}(MV) = M.\frac{dV}{dt} = M.\frac{d^2R}{dt^2}$$

$$F_e = \frac{dP}{dt} = \frac{d}{dt}(MV)$$

when $F_e = 0$,

$$P = MV = \sum_i m_i V_i = \text{const.}$$

Centre of Mass - Frame of reference

Inertial frame - where no external force acting on system or $F^e = 0$.

Conservation of Angular Momentum

$J_1, J_2, \cdots J_N$. angular momenta of particles

$$J = J_1 + J_2 + \cdots J_N$$

$$J = (r_1 \times P_1) + (r_2 \times P_2) + \cdots (r_N \times P_N)$$

$$= \sum_{i=1}^{N} r_i \times P_i$$

Also we know

$$\tau = \frac{dJ}{dt} \qquad (\because \dot{r}_i \times P_i = \dot{r}_i \times m\dot{r}_i = 0.$$

$$= \sum_{i=1}^{N} (r_i \times \dot{P}_i)$$

$$= \sum_i (r_i \times F_i)$$

$$F_i = F_i^e + \sum_{j=1}^{N} F_{ij}$$

producting $\vec{r}\times$ & summation

$$\sum_i (r_i \times F_i) = \sum_i (r_i \times F_i^e) + \sum_i \sum_j (r_i \times F_{ij})$$

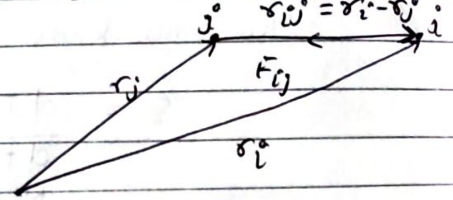
$$\sum_i \sum_j (r_i \times F_{ij}) = r_i \times F_{ij} + r_j \times F_{ji}$$

$F_{ij} = - F_{ji}$ Newtons 3rd Law.

$$\sum_i \sum_j (r_i \times F_{ij}) = r_i \times F_{ij} - r_j \times F_{ij}$$

$$= (r_i - r_j) \times F_{ij}$$

$$\sum_i (r_i \times F_i^e) = \sum_i (r_i \times F_i^e) = \tau^e.$$



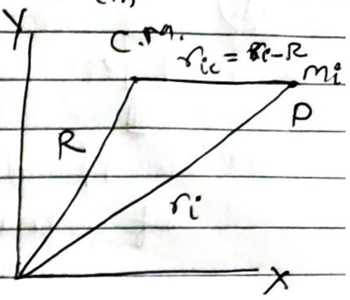
The $\vec{r}_{ij} \times \vec{F}_{ij} = 0.$

$$\tau^e = \frac{dJ}{dt}$$

if $\tau^e = 0$

$$J = J_1 + J_2 + \text{---} J_N = \text{constant}$$

Relation b/w J and J_{cm}



$$J = J_{cm} + R \times P$$

where

$$J_{cm} = \sum_i (r_{ic} \times M_i V_{ic})$$

Conservation of energy

$$W_{12} = \int_1^2 F . dr$$ for one particle

for whole system

$$W_{12} = \sum_{i=1}^{N} \int_1^2 F_i . dr_i$$

$$= \sum_i \int_1^2 F_i^e . dr_i + \sum_i \sum_j \int_1^2 F_{ij} . dr_i$$

(a) Kinetic energy

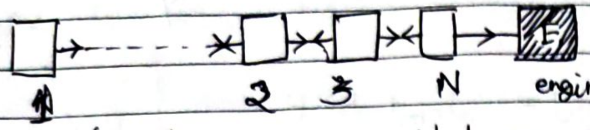
$$T = \sum_i \frac{1}{2} m_i v_{ic}^2 + \frac{1}{2} M V^2$$

(b) Potential energy

$$V = \sum_i V_i + \frac{1}{2} \sum_i \sum_j V_{ij} \quad i \neq j$$

Some important formulas.

(1)



if there are N boxes each of mass M.
then acceleration.

$$a = \frac{F}{NM} \Rightarrow Ma = F/N$$

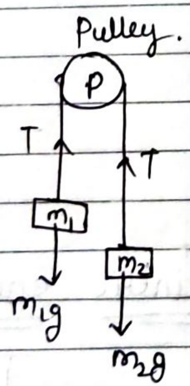
and on each box net force will be F/N

and forward force on n^{th} box will be

$$nF/N$$

(2) $m_1 a = T - m_1 g.$

$m_2 g - T = m_2 a.$



$$T = m_1\left(g + \frac{m_2 - m_1}{m_1 + m_2}\right) =$$

$$= \frac{2 m_1 m_2}{m_1 + m_2} \cdot g$$ where $g = 9.8\ m/s^2$

(3) For conservative force field

curl $F = 0$

because

$$\oint F \cdot dr = \iint \text{curl } F \cdot ds = 0$$

$$\text{curl } F = 0$$

$$\nabla \times F = 0$$

Definition of Central force :-

If a force acts on a particle such a way that it is always directed towards or away from a fixed points and its magnitude depends only upon the distance (r) from the point, then this force is called a central force.

$$F = F(r)\,\hat{r} = F(r)\,\bar{r}/r$$

gravitational $F(r) = -\frac{G m_1 m_2}{r^2}$

& coulomb $F(r) = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$.

(4) when particle moves under a central force field

(i) we know

$$\tau = \frac{dJ}{dt}$$

$$= \frac{dJ}{dt} = \bar{r} \times F(r)\frac{\bar{r}}{r}$$

$$\tau = \frac{dJ}{dt} = 0$$

$$\boxed{J = \text{conserved}}$$

(ii)

$$J = r \times p$$

$$r \cdot J = r \cdot (r \times p)$$

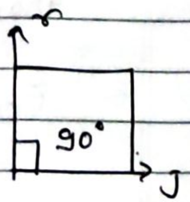
$$= (r \times r) \cdot p$$

$$r \cdot J = 0$$

$$r J \cos\theta = 0$$

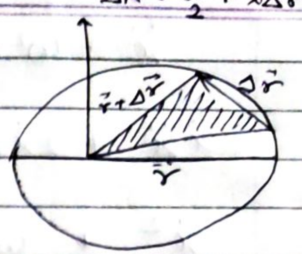
$$\theta = 90^\circ$$

so



so motion takes place in a plane.

$$\Delta A = \frac{1}{2} r \times \Delta r$$



$$\frac{dA}{dt} = \frac{1}{2} r \times v$$

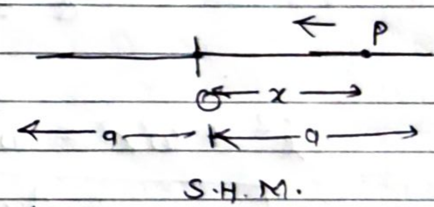
$$\frac{dA}{dt} = \frac{J}{2m}$$

areal velocity.

J is constant means $\frac{dA}{dt}$ = also constant.

(5) Simple harmonic Oscillator

now



$$m\frac{d^2x}{dt^2} + Cx = 0$$

C is force constant

$$F = m\ddot{x} = -Cx$$

$$\Rightarrow m\ddot{x} + Cx = 0 \qquad \omega = \sqrt{\frac{C}{m}}$$

$$m\frac{d^2x}{dt^2} + Cx = 0$$

$$\frac{d^2x}{dt^2} + \frac{C}{m}x = 0$$

$$\ddot{x} + \omega^2 x = 0$$

$$\boxed{\text{acceleration} = \ddot{x} = -\omega^2 x}$$

$$\ddot{x} + \omega^2 x = 0$$

$$2\dot{x}\ddot{x} + 2\dot{x}x\omega^2 = 0$$

$$\dot{x}^2 + \omega^2 x^2 = A$$ (integration constant)

when displacement is maximum

means $x = a$

$\dot{x}, \ddot{x} = 0$

$$0 + \omega^2 a^2 = A$$

$$A = \omega^2 a^2$$

$$\dot{x}^2 + \omega^2 x^2 = \omega^2 a^2$$

$$\boxed{\dot{x} = \frac{dx}{dt} = \omega\sqrt{a^2 - x^2}}$$

$$\boxed{x = a \sin(\omega t + \phi)}$$